Chapter 5: Strain-Hardening of Metals

Prof. Wenjea J. Tseng (曾文甲)
Department of Materials Engineering
National Chung Hsing University
wenjea@dragon.nchu.edu.tw

Reference: W.F. Hosford (Cambridge, 2010)

Strain Hardening

- Strain hardening or work hardening describes the increase of stress necessary to continue deformation at any stage of plastic strain.
- Various true stress-strain curves for work hardening:
  - Elastic, perfectly plastic material
    \[ \sigma = Y \]
  - Linear work-hardening
    \[ \sigma = Y + A\varepsilon \]
  - Power-law work hardening
    \[ \sigma = K\varepsilon^n \]
  - Modifications of the power-law relationship
    \[ \sigma = \sigma_0 [1 - \exp(-A\varepsilon)] \]

The strain hardening tends to increase the load-carrying capacity of the specimen as deformation increases.
Strain Hardening

- Typical values of $n$ and $K$ are listed in the following Table.

<table>
<thead>
<tr>
<th>Material</th>
<th>Strength coefficient, $K$ (MPa)</th>
<th>Strain-hardening exponent, $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>low carbon steels</td>
<td>525 to 670</td>
<td>0.20 to 0.25</td>
</tr>
<tr>
<td>HSLA steels</td>
<td>650 to 900</td>
<td>0.15 to 0.20</td>
</tr>
<tr>
<td>austenite stainless</td>
<td>400 to 500</td>
<td>0.40 to 0.55</td>
</tr>
<tr>
<td>copper</td>
<td>420 to 480</td>
<td>0.35 to 0.50</td>
</tr>
<tr>
<td>70/30 brass</td>
<td>525 to 750</td>
<td>0.45 to 0.60</td>
</tr>
<tr>
<td>aluminum alloys</td>
<td>400 to 550</td>
<td>0.20 to 0.30</td>
</tr>
</tbody>
</table>


Mechanically stronger metals are in general with a smaller strain-hardening exponent, and vice versa.

Power-law Approximation

- The most commonly used expression for strain hardening is the simple power law.

$$ \sigma = K \varepsilon^n $$

- As $n$ varies, the shape of the true stress-strain curve also alters (see figure). If $n$ is low, the work-hardening rate is initially high but the rate decreases rapidly with strain. On the other hand, with a high $n$, the initial work hardening is less rapid but continues to high strains.
Power-law Approximation

- If we take $\ln$ on both sides of the power-law equation,

$$\ln \sigma = \ln K + n \ln \varepsilon$$

(Note: $\sigma$, $\varepsilon$ in true stress and true strain)

- In mathematical form, $n$ can also be determined by

$$n = \frac{d(\ln \sigma)}{d(\ln \varepsilon)} = \frac{(1/\sigma)d\sigma}{(1/\varepsilon)d\varepsilon} = \frac{\varepsilon}{\sigma} \cdot \frac{d\sigma}{d\varepsilon}$$

$n$ is the slope of the linear portion of the curve, and can hence be experimentally determined. $K$ can be found by inserting $\varepsilon = 1$.

Instability in Tension as Neck Forms

- Metals often undergo strain hardening as deformation increases. This is opposed by the gradual decrease in the cross-sectional area of the specimen as it elongates. Necking begins at maximum load ($F_{\text{max}}$), where the increase in stress due to decrease in cross-sectional area becomes greater than the increase in the load-carrying capability due to strain hardening. This leads to instability of localized deformation defined by $dF = 0$.

$dF = 0$ as $F = F_{\text{max}}$

This viewgraph tries to relate the strain hardening of metals in the true stress-strain curve to the engineering stress-strain behavior.
Instability in Tension as Neck Forms

- \( F = \sigma A; \) therefore, \( dF = \sigma dA + Ad\sigma = 0 \) at \( F = F_{\text{max}} \)

The volume of specimen is conserved, \( dV = d(AL) = 0, \) hence

\[
AdL + LdA = 0
\]

\[
\frac{dA}{A} = \frac{dL}{L} = -d\varepsilon
\]

Substituting this relation to the \( dF = 0 \) equation, we obtain

\[
dF = \sigma(-Ad\varepsilon) + Ad\sigma = 0
\]

\[
\therefore -\sigma d\varepsilon + d\sigma = 0 \quad \text{Hence}, \quad \frac{d\sigma}{d\varepsilon} = \sigma
\]

This equation simply states that the point of necking at maximum load can be obtained from the true stress-strain curve by finding the point where the rate of strain hardening equals the stress (see figure).

Instability in Tension as Neck Forms

- The above derivation for neck formation on true stress-strain curve can be extended if engineering strain is used.

\[
\frac{d\sigma}{de} = \frac{d\sigma}{de} \frac{de}{de} = \frac{d\sigma}{de} \frac{de}{dL} \frac{dL}{L} = \frac{d\sigma}{de} \frac{L}{de} = \frac{d\sigma}{de} \frac{(1+\varepsilon)}{\varepsilon} = \sigma
\]

Hence,

\[
\frac{d\sigma}{de} = \frac{\sigma}{1+\varepsilon}
\]

Considère’s construction for the determination of the point of maximum load (\( \sigma_{\text{UTS}} \)).

Can find \( \sigma_{\text{UTS}} \).
Mechanisms for Strain Hardening

- What is the cause for strain hardening in metals? The main cause is by dislocations interacting with each other and with barriers (e.g., dislocations pile up at grain boundaries, twins, inclusions, etc.) which impede their motion through the crystal lattice.
- The cause for strain hardening is often conceived as a generation of “back stress” when dislocation interaction and dislocation pile up against a barrier become pronounced to oppose the motion of additional dislocations along the slip plane.
- Bauschinger effect: The lowering of the yield stress when deformation in one direction is followed by deformation in the opposite direction.

Summary

- Mathematical approximation used mostly frequently in modeling the true stress-strain behavior of strain-hardened metals in the plastic region is the power-law equation, i.e.,

$$
\sigma = K \varepsilon^n
$$

- With a higher $n$, the initial work hardening is less rapid than that of the lower $n$ metals but the stress required to further deformation would continue to high strains. The exponent can be determined from $\ln(\sigma)$-$\ln(\varepsilon)$ dependence

$$
n = \frac{d(\ln \sigma)}{d(\ln \varepsilon)} = \frac{(1/\sigma) d\sigma}{(1/\varepsilon) d\varepsilon} = \varepsilon \frac{d\sigma}{\sigma d\varepsilon}
$$
Power-law Approximation

- The true stress-strain curve for various $n$ values.

Necking

- The condition for necking in a tension test is met when the true stress equals the slope in a true stress-strain curve.