Mechanical Properties of Materials

Chapter 5: Strain-Hardening of Metals

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Reference: W.F. Hosford (Cambridge, 2010)
G.E. Dieter, Mechanical Metallurgy (McGraw-Hill, 1988)

Strain Hardening

The strain hardening tends to increase the load-carrying capacity of the specimen as deformation increases.

- Strain hardening or work hardening describes the increase of stress necessary to continue deformation at any stage of ***plastic*** strain.
- Various ***true*** stress-strain curves for work hardening:

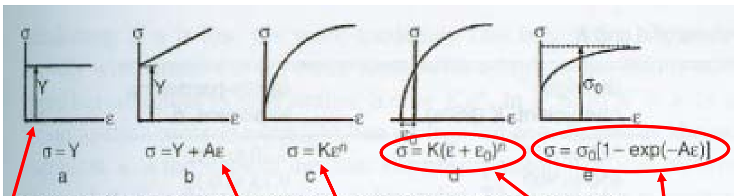


Figure 5.1. Mathematical approximations of the true stress–strain curve.

Elastic, perfectly plastic material

$$\sigma = Y$$

Y: tensile yield strength

Linear work-hardening

$$\sigma = Y + A\varepsilon$$

Power-law work hardening

$$\sigma = K\varepsilon^n$$

Modifications of the power-law relationship

Strain Hardening

- Typical values of n and K are listed in the following Table.

Material	Strength coefficient, *K* (MPa)	Strain-hardening exponent, *n*
low-carbon steels	525 to 575	0.20 to 0.23
HSLA steels	650 to 900	0.15 to 0.18
austenitic stainless	400 to 500	0.40 to 0.55
copper	420 to 480	0.35 to 0.50
70/30 brass	525 to 750	0.45 to 0.60
aluminum alloys	400 to 550	0.20 to 0.30

Note: From various sources including W. F. Hosford and R. M. Caddell, *Metal Forming: Mechanics and Metallurgy*, Prentice–Hall, 1983.

Mechanically stronger metals are in general with a smaller strain-hardening exponent, and vice versa.

Power-law Approximation

- The most commonly used expression for strain hardening is the simple power law.

$$\sigma = K\varepsilon^n$$

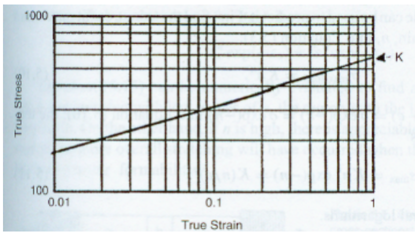
- As n varies, the shape of the true stress-strain curve also alters (see figure). If *n* is low, the work-hardening rate is initially high but the rate decreases rapidly with strain. On the other hand, with a high *n*, the initial work hardening is less rapid but continues to high strains.

Power-law Approximation

- If we take *ln* on both sides of the power-law equation,

$$\ln\sigma = \ln K + n\ln\varepsilon$$

(Note: σ, ε in **true** stress and **true** strain)



n is the slope of the linear portion of the curve, and can hence be experimentally determined. K can be found by inserting ε = 1.

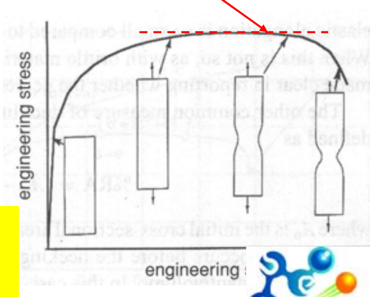
- In mathematical form, *n* can also be determined by

$$n = \frac{d(\ln\sigma)}{d(\ln\varepsilon)} = \frac{(1/\sigma)d\sigma}{(1/\varepsilon)d\varepsilon} = \frac{\varepsilon}{\sigma}\cdot\frac{d\sigma}{d\varepsilon}$$

Instability in Tension as Neck Forms

- Metals often undergo strain hardening as deformation increases. This is opposed by the gradual decrease in the cross-sectional area of the specimen as it elongates. Necking begins at maximum load (F_{max}), where the increase in stress due to decrease in cross-sectional area becomes greater than the increase in the load-carrying capability due to strain hardening. This leads to *instability* of localized deformation defined by dF = 0.

$dF = 0$ as $F = F_{max}$



This viewgraph tries to relate the strain hardening of metals in the true stress-strain curve to the engineering stress-strain behavior.

Instability in Tension as Neck Forms

- $F = \sigma A$; therefore, $dF = \sigma dA + A d\sigma = 0$ at $F = F_{max}$
  The volume of specimen is conserved, $dV = d(AL) = 0$, hence $AdL + LdA = 0$

$$\frac{dA}{A} = -\frac{dL}{L} = -d\varepsilon$$

Substituting this relation to the dF = 0 equation, we obtain

$$dF = \sigma(-Ad\varepsilon) + Ad\sigma = 0$$

$$\therefore -\sigma d\varepsilon + d\sigma = 0 \qquad \text{Hence,} \quad \frac{d\sigma}{d\varepsilon} = \sigma$$

This equation simply states that the point of necking at maximum load can be obtained from the true stress-strain curve by finding the point where the rate of strain hardening equals the stress (see figure).

Instability in Tension as Neck Forms

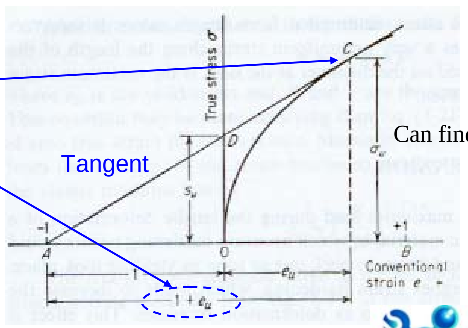
- The above derivation for neck formation on true stress-strain curve can be extended if engineering strain is used.

$$\frac{d\sigma}{d\varepsilon} = \frac{d\sigma}{d\varepsilon_e}\frac{d\varepsilon_e}{d\varepsilon} = \frac{d\sigma}{d\varepsilon_e}\frac{dL/L_o}{dL/L} = \frac{d\sigma}{d\varepsilon_e}\frac{L}{L_o} = \frac{d\sigma}{d\varepsilon_e}(1+\varepsilon_e) = \sigma$$

Hence,

$$\frac{d\sigma}{d\varepsilon_e} = \frac{\sigma}{1+\varepsilon_e}$$

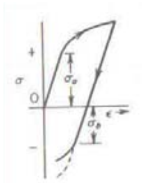
Considère's construction for the determination of the point of maximum load (σ_{UTS}).



Can find σ_{uts}

Mechanisms for Strain Hardening

- What is the cause for strain hardening in metals? The main cause is by dislocations interacting with each other and with barriers (e.g., dislocations pile up at grain boundaries, twins, inclusions, etc.) which impede their motion through the crystal lattice.
- The cause for strain hardening is often conceived as a generation of "back stress" when dislocation interaction and dislocation pile up against a barrier become pronounced to oppose the motion of additional dislocations along the slip plane.
- Bauschinger effect: The lowering of the yield stress when deformation in one direction is followed by deformation in the opposite direction.



Summary

- Mathematical approximation used mostly frequently in modeling the true stress-strain behavior of strain-hardened metals in the plastic region is the power-law equation, i.e.,

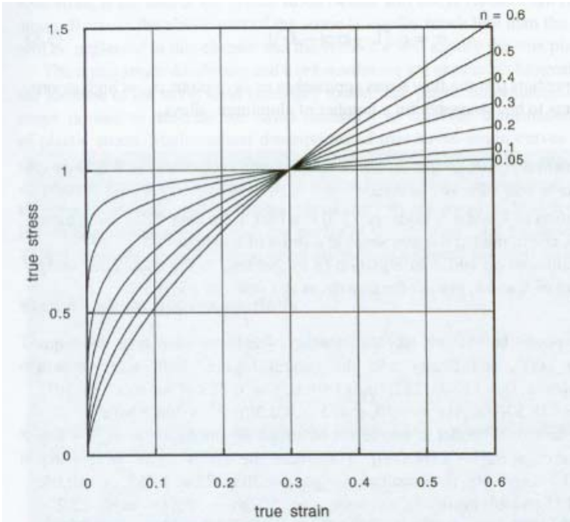
$$\sigma = K\varepsilon^n$$

- With a higher *n*, the initial work hardening is less rapid than that of the lower *n* metals but the stress required to further deformation would continue to high strains. The exponent can be determined from ln(σ)-ln(ε) dependence

$$n = \frac{d(\ln\sigma)}{d(\ln\varepsilon)} = \frac{(1/\sigma)d\sigma}{(1/\varepsilon)d\varepsilon} = \frac{\varepsilon}{\sigma}\cdot\frac{d\sigma}{d\varepsilon}$$

Power-law Approximation

- The true stress-strain curve for various *n* values.



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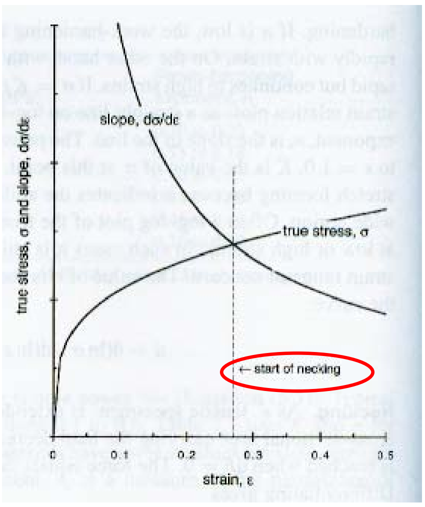
Necking

- The condition for necking in a tension test is met when the true stress equals the slope in a true strss-strain curve.



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